Semantics of CTL

base case: If $p \in AP$, then p is a state formula.

inductive cases: if α, β are state formulas and ϕ is a path formula, then the following are state formulas:

$\neg\alpha$, $\alpha \vee \beta$, $E\phi$, $A\phi$

Syntax of Path Formulas:

If α is a state formula and ϕ and ψ are path formulas, then the following are path formulas:

α, $\neg\phi$, $(\phi \vee \psi)$, $X\phi$, $F\phi$, $G\phi$, $(\phi U \psi)$

Semantics of State Formulas:

$(T , s) \models p \Leftrightarrow p \in L(s)$

$(T , s) \models \neg\alpha \Leftrightarrow (T , s)\ 6{\models}\ \alpha$

$(T , s) \models (\alpha \vee \beta) \Leftrightarrow (T , s) \models \alpha$ or $(T , s) \models \beta$

$(T , s) \models E\phi \Leftrightarrow$ there exists path π, $\pi[0] = s$, $(T , \pi) \models \phi$

$(T , s)\ \ A\phi \Leftrightarrow$ for all π such that $\pi[0] = s$, $(T , \pi)\ \phi$

For α a state formula, $(T , \pi)\ \ \alpha \Leftrightarrow (T , \pi[0])\ \alpha$

Semantics of Path Formula : (same as in LTL)

$(T , \pi) \models \neg\alpha$ iff $(T , \pi)\ 6{\models}\ \alpha$

$(T , \pi) \models (\alpha \vee \beta)$ iff $(T , \pi) \models \alpha$ or $(T , \pi) \models \beta$

$(T , \pi) \models X\alpha$ iff $\pi^1 \models \alpha$

$(T , \pi) \models G\alpha$ iff $\forall i \geq 0\ (T , \pi i) \models \alpha$

$(T , \pi) \models F\alpha$ iff $\exists i \geq 0\ (T , \pi i) \models \alpha$

$(T , \pi) \models (\alpha U \beta)$ iff $\exists i \geq 0\ ((T , \pi i) \models \beta \wedge \forall j < i\ (T , \pi j) \models \alpha)$

Syntax of CTL

The language of well-formed formulas for CTL is generated by the following grammar:

$$\begin{aligned}\phi ::= & \bot \mid \top \mid p \mid (\neg\phi) \mid (\phi \wedge \phi) \mid (\phi \vee \phi) \mid (\phi \Rightarrow \phi) \mid (\phi \Leftrightarrow \phi) \\ \mid \quad & \mathrm{AX}\,\phi \mid \mathrm{EX}\,\phi \mid \mathrm{AF}\,\phi \mid \mathrm{EF}\,\phi \mid \mathrm{AG}\,\phi \mid \mathrm{EG}\,\phi \mid \mathrm{A}\,[\phi\,\mathrm{U}\,\phi] \mid \mathrm{E}\,[\phi\,\mathrm{U}\,\phi]\end{aligned}$$

where p ranges over a set of atomic formulas. It is not necessary to use all connectives – for example, $\{\neg, \wedge, \mathrm{AX}, \mathrm{AU}, \mathrm{EU}\}$ comprises a complete set of connectives, and the others can be defined using them.

- $\mathbf{A}$ means 'along All paths' *(inevitably)*
- $\mathbf{E}$ means 'along at least (there Exists) one path' *(possibly)*

For example, the following is a well-formed CTL formula:

$$\mathrm{EF}\,(\mathrm{EG}\,p \Rightarrow \mathrm{AF}\,r)$$

The following is not a well-formed CTL formula:

$$\mathrm{EF}\,\left(r\,\mathrm{U}\,q\right)$$

The problem with this string is that $\mathbf{U}$ can occur only when paired with an $\mathbf{A}$ or an $\mathbf{E}$.

CTL uses atomic propositions as its building blocks to make statements about the states of a system. These propositions are then combined into formulas using logical operators and temporal operators.